LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FOURTH SEMESTER – **APRIL 2024**

UST 4501 – ESTIMATION THEORY

Date: 06-04-2024 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions
1.	**Define the following** **(5 x 1 = 5)**
a)	Consistency
b)	Most efficient estimator
c)	Minimum variance unbiased estimator
d)	Likelihood function
e)	Confidence limits
2.	**MCQ- Choose the correct option** **(5 x 1 = 5)**
a)	Let T_n be an estimator of θ. If $E(T_n) = \theta$, then i) T_n is a sufficient estimator of θ ii)T_n is an unbiased estimator of θ iii)T_n is a consistent estimator of θ iv) T_n is an efficient estimator of θ
b)	Rao-Blackwell theorem enables us to Minimum variance unbiased estimate through i) Complete Statistics ii) Efficient statistics iii) Sufficient Statistics iv) Unbiased Estimator
c)	Let$E(T_1) = \theta = E(T_2)$, where T_1 and T_2 are the linear functions of the sample observations. If $V(T_1) \leq V(T_2)$ then: i)T_1 is an unbiased linear estimator ii) T_1 is the best linear unbiased estimator iii) T_1 is a consistent linear unbiased estimator iv) T_1 is a consistent best linear unbiased estimator
d)	The maximum likelihood estimates, which are obtained by maximizing the function of Joint Density of random variables are generally: i)Unbiased and inconsistent ii)Unbiased and consistent iii)Consistent and invariant iv)Invariant and unbiased
e)	The random interval $\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{\chi^2_{n-1,\alpha/2}} , \frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{\chi^2_{n-1,1-\alpha/2}}\right)$ gives the confidence interval for i) Mean μ of a normal population when σ^2 is known

	ii) Mean μ of a normal population when σ^2 is unknown iii) Variance σ^2 of a normal population when is μ known iv) Variance σ^2 of a normal population when is μ unknown
	SECTION A - K2 (CO1)
	Answer ALL the Questions
3.	**True or False** **(5 x 1 = 5)**
a)	Unbiased estimator is always function of complete sufficient statistic
b)	An unbiased estimator T of $\gamma(\theta)$ for which Cramer Rao lower bound of V (θ) is attained, is called a minimum variance bound estimator.
c)	Let $X_1, X_2, \ldots X_n$ be a random sample from N(0,θ), then $T=X_1$ is a complete statistic for θ.
d)	Method of moment estimators are always unique.
e)	An Interval Estimate is an estimate of the range of possible values for a population parameter.
4.	**Fill in the blanks** **(5 x 1 = 5)**
a)	Let $\{T_n\}$ be sequence of estimators such that for all , $\theta \in \Theta$, (i)------------------------ and (ii) ------------------- then , T_n is a consistent estimator of γ(θ)
b)	Let $x_1, x_2, \ldots x_n$ be a random sample from a density function f(x,θ). A statistic $T=t(x_1,x_2,\ldots x_n)$ is said to be sufficient statistics if the conditional distribution of $x_1,x_2,\ldots x_n$ given T is ---------------------
c)	If T is unbiased for $\gamma(\theta)$and it has the smallest variance among the class of all unbiased estimators of $\gamma(\theta)$, then it is said to be--------------------------
d)	The equation of Maximum Likelihood Estimators is given by -------------------
e)	The term (1-α) refers to the-------------------

SECTION B - K3 (CO2)

Answer any TWO of the following	**(2 x 10 = 20)**
5.	Prove invariance property of consistent estimators
6.	State and prove Cramer-Rao inequality
7.	Prove Lehmann-Scheffe theorem
8.	Prove that, If T_1 is an MVUE of $\gamma(\theta), \theta \in \Theta$ and T_2 is any other unbiased estimator of $\gamma(\theta)$ with efficiency $e = e_\theta$, then the correlation between T_1 and T_2 is given by $\rho = \sqrt{e}$.

SECTION C – K4 (CO3)

Answer any TWO of the following	**(2 x 10 = 20)**
9.	State and prove Rao-Blackwell Theorem
10.	Obtain Maximum Likelihood Estimators of the parameters of the normal distribution.
11.	If T_1 is an Minimum Variance Unbiased estimator of γ(θ) and T_2 is any other unbiased estimator of γ(θ) with efficiency e<1, then prove that no unbiased linear combination of $T_1$and T_2 can be an Minimum Variance Unbiased estimator of γ(θ)
12.	Obtain 100(1-α)% confidence limits for the parameter λ of a Poisson distribution.

SECTION D – K5 (CO4)

Answer any ONE of the following	**(1 x 20 = 20)**
13.	a) X_1, X_2, X_3 is a random sample of size three from a population with mean μ and variance σ^2 . T_1,

	T_2, T_3 are the estimators used to estimate mean value μ where $T_1 = X_1 - X_2 + X_3$; $T_2 = 2X_1 - X_2 + 3X_3$; $T_3 = 1/3\ [(\lambda X_1 + X_2 + X_3)/3]$ answer the following (10) (i) T_1 and T_2 are unbiased? (ii) Find λ and check T_3 is unbiased and consistent estimator (iii) which is the best estimator? b) Let $X_1, X_2, \ldots X_n$ be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimators μ and σ^2 (10)
14.	(a) Obtain 100(1-α)% confidence interval for ratio of variances of two normal populations. (10) (b) Given one observation from a population with probability density function $$f(x,\theta) = \frac{2}{\theta^2}(\theta - x), 0 \le x \le \theta$$ Obtain 100(1-α)% confidence interval for θ. (10)

SECTION E – K6 (CO5)

Answer any ONE of the following	**(1 x 20 = 20)**
15.	For the double Poisson distribution $P(x) = \frac{1}{2}\frac{e^{-m_1} m_1^{\ x}}{x!} + \frac{1}{2}\frac{e^{-m_2} m_2^{\ x}}{x!}$; x = 0,1,2…. obtain the estimates for m_1 and m_2 by the method of moments.
16.	a) Find the MLE of the parameter of the exponential distribution with mean λ. Check whether it is an unbiased estimator (10) b) State and prove Factorization Theorem (10)

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